DETERMINATION OF THE LOCAL COEFFICIENTS OF HEAT TRANSFER TO A LIQUID IN SQUARE CHANNELS

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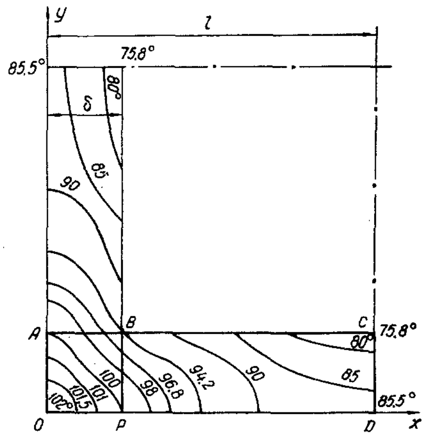
An analytical method is developed for determining the temperatures and heat-transfer coefficients at the inner perimeter of channels of square cross section with uniform volume heat release in the walls in the case of a given temperature distribution over the outer perimeter of the channels.

Tubes of rectangular cross section find widespread application in various heat exchangers. For turbulent



Temperature field in the walls of a square channel calculated from formula (6) for l = = 3.05 mm, δ = 0.7 mm, h = α/λ_w = 0.398 1/mm, q_v/λ_w = 30.6 deg/mm^2, t_l = 85.5° C, t_{fl} = 16.8° C.

flow, the mean heat transfer in channels on noncircular cross section is usually calculated to an accuracy of 10% from the formulas for circular tubes, taking the hydraulic equivalent diameter as the characteristic dimension. In the case of high thermal loads and internal heat release in the channel walls, a corner past which the fluid flows at a low rate experiences the highest thermal stress. The problem of determining the local heat-transfer coefficients at the corners is therefore of practical importance.

Deissler and Teilor [1] have proposed a method of calculating the distribution of the shear stresses, temperatures, and heat-transfer coefficients over the perimeter of rectangular channels. Their calculations were based on a semiempirical turbulence theory and on the assumption that a universal velocity distribution law is valid in the case under consideration. The heat-transfer coefficients were assumed to have zero values at a corner. The experimental values of the resistance coefficients averaged over the cross section of a square channel proved to exceed the theoretical values [1] by 12% [2], while the heat-transfer coefficients at Re $\geq 5 \cdot 10^4$ exceeded the theoretical value by 10% [3]. According to the authors of [2], these discrepancies may be attributed to the fact that the theory [1] neglects the secondary currents which act to increase resistance and heat transfer in the corner areas.

Graphs of the temperature and heat-flux distribution over the perimeter of rectangular channels were plotted analytically in [4] on the basis of Nikuradze's [5] empirical distribution of shear stresses at the channel wells for isothermal flow. Naturally, for this case, the region of high temperatures and low thermal fluxes at the corners is appreciably smaller than in [1].

At the same time, we are not aware of any experiments aimed at a direct determination of the temperatures and heat-transfer coefficients along the perimeter of rectangular channels. An exception is [6], where the temperature distribution over the inner perimeter of a rectangular channel was measured for mercury flow. In these experiments, the measured corner temperatures were only slightly higher than the temperatures at the middle of the walls. It was also observed that the surface temperature distribution equalizes with increasing Peclet number.

Closely related to our investigation is the careful experimental study performed by Eckert and Irvine [7], in which the local heat-transfer coefficients were determined for air heated in an isoceles triangular channel with vertex angle of 11.5°. The graphs obtained in [7] reveal that at the base of the triangle and at the 84.25° angle itself, the heat-transfer coefficients are maximum and practically constant. In this region, the ratio of the local heat-transfer coefficients to the mean cross-sectional coefficients is roughly equal to 2.1. The authors of [7] found that the mean heat-transfer coefficients in the channel studied were approximately half their value calculated from a formula for circular tubes.

Thus, the local heat-transfer coefficients at a nearly right-angled corner and in its proximity are roughly equal to those in circular tubes. This result [7] disagrees qualitatively with the statements in [1, 5], according to which the heat-transfer coefficients approach zero at a corner.

An experimental determination of the temperatures and heat-transfer coefficients along the perimeter of square channels is thus undoubtedly of interest. It may be noted that the importance and actuality of this problem was emphasized in [8].

Usually, the procedure employed in studying heat transfer for fluid flows in channels consists in measuring the temperature of the outer surface, calculating the temperature gradients in the wall, and then calculating the coefficients of heat transfer to the fluid. For circular tubes, the problem of determining the temperature gradients in the walls has been solved analytically. For channels of complex cross section, the temperature gradients in the walls are conventionally determined by numerical methods, for example, with the aid of finite-difference schemes [4].

In the following, a procedure is described for obtaining an analytical solution to the problem of determining the local heat-transfer coefficients and temperature distributions along the inner perimeter of tubes of square cross section with uniform volume heat release in the walls, for a given temperature distribution over the outer perimeter of the channel. It is assumed that there is no heat flow in the axial direction along the walls, i.e., the discussion is limited to the two-dimensional problem. The heat-transfer coefficient of the wall material, taken for a certain mean wall temperature, is postulated to be constant. The error introduced by this simplification is relatively small even for materials of low thermal conductivity such as stainless steel, characterized by large wall temperature gradients at high thermal loads. The outer surface of the tube is assumed to be adiabatic, which in most cases corresponds to the test conditions, since usually the heat release to the atmosphere does not exceed 5%. We understand the temperature of the fluid (t_{fl}) to be the mean cross-sectional flow temperature.

Let us examine the cross section of a square channel (see the figure). In view of the symmetry with respect to axes passing through the middle of the sides, the problem is to be solved for the rectangular element ACDO.

By introducing the notation $\theta = t - t_{fl}$, the heat equation takes the form

$$\frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2} = -\frac{q_v}{\lambda}. \tag{1}$$

The boundary conditions for the element ACDO are

$$\left.\frac{\partial \theta}{\partial x}\right|_{x=0} = 0, \quad \left.\frac{\partial \theta}{\partial y}\right|_{y=0} = 0, \quad \left.\frac{\partial \theta}{\partial x}\right|_{x=l} = 0, \tag{2}$$

$$\frac{\partial \theta}{\partial y} + h(x)\theta|_{y=\delta} = \begin{cases} Q(x), & 0 \leqslant x \leqslant \delta, \\ 0, & \delta \leqslant x \leqslant l. \end{cases} \tag{3}$$

The condition for the intercept BP may be written in the same form as for BA:

$$\frac{\partial \theta}{\partial x} + h(y)\theta|_{x=\delta} = Q(y), \quad 0 \leqslant y \leqslant \delta, \tag{4}$$

where Q(x) and Q(y) are certain functions, and $h(x) = \alpha(x)/\lambda$; $h(y) = \alpha(y)/\lambda$. We write the functions h(x) and h(y), which characterize (except on the intercepts AB and BP, where they are fictitious) the variation of the heat-transfer coefficients along the perimeter, in the form

$$h(x) = a_0 + \sum_{k=1}^{\infty} a_k \varphi_k(x),$$
$$h(y) = a_0 + \sum_{k=1}^{\infty} a_k \varphi_k(y), \tag{5}$$

where $a_0, \ldots, a_k$ are certain unknown coefficients, while $\varphi_k(x)$ and $\varphi_k(y)$ are certain functions which may be expressed, for example, by polynomials.

By solving the problem with the aid of eigenfunctions, an expression is obtained for the temperature distribution in the wall, in the case where the heat-transfer coefficient is constant along the perimeter:

$$\theta(x, y) = \frac{1}{hl}\left(Q_0 + \frac{q_v l\delta}{\lambda} + \frac{hq_v l\delta^2}{2\lambda}\right) - \frac{q_v y^2}{2\lambda} + \sum_{n=1}^{\infty} \frac{2Q_n \operatorname{ch}\frac{\pi n}{l} y \cos\frac{\pi n}{l} x}{\pi n \operatorname{sh}\frac{\pi n}{l}\delta + hl \operatorname{ch}\frac{\pi n}{l}\delta}, \tag{6}$$

where $Q_0, Q_1, \ldots, Q_n$ are arbitrary constants.

Then, for an arbitrary distribution of the heat-transfer coefficients over the perimeter, we have

$$\theta(x, y) = F_0 - \frac{q_v y^2}{2\lambda} + \sum_{n=1}^{\infty} F_n \operatorname{ch}\frac{\pi n}{l} y \cos\frac{\pi n}{l} x, \tag{7}$$

where $F_0, F_1, \ldots, F_n$ are certain unknown coefficients that depend on h(x, y), q_v/λ, δ, and 1.

It should be noted that the eigenvalues and eigenfunctions for an arbitrary distribution of the heat-transfer coefficients over the perimeter are the same as in the case of a constant heat-transfer coefficient, namely: eigenvalues $\nu_n = (\pi n/l^2$; eigenfunctions $X_n = \cos(\pi n/l)x$, where n = 1, 2, ...; zero eigenvalue $\nu_0 = 0$; zero eigenfunction $X_0 = 1$.

Equation (7) satisfies all boundary conditions except conditions (3) and (4) which, with allowance for (5), may be written in the form

$$\frac{\partial \theta}{\partial x} + \theta\left[a_0 + \sum_{k=1}^{\infty} a_k \varphi_k(y)\right]\Bigg|_{x=\delta} = Q(y), \quad 0 \leqslant y \leqslant \delta, \tag{8}$$

$$\frac{\partial \theta}{\partial y} + \theta\left[a_0 + \sum_{k=1}^{\infty} a_k \varphi_k(x)\right]\Bigg|_{\delta=y} = \begin{cases} Q(x), & 0 \leqslant x \leqslant \delta, \\ 0, & \delta \leqslant x \leqslant l. \end{cases} \tag{9}$$

Having made (7) satisfy condition (8), we multiply the obtained equation successively first by the eigenfunctions $\cos(\pi/l)y, \ldots, \cos(\pi m/l)y$, and then by $Y_0 = 1$, and integrate over y from 0 to δ. The resulting system of m equations has the form

$$\sum_{n=1}^{\infty} F_n\left(a_0 \cos\frac{\pi n}{l}\delta - \frac{\pi n}{l}\sin\frac{\pi n}{l}\delta\right) \times \int_0^{\delta} \operatorname{ch}\frac{\pi n}{l} y \cos\frac{\pi m}{l} y dy + a_0\left(F_0 \int_0^{\delta} \cos\frac{\pi m}{l} y dy - \right. \tag{10}$$

$$-\frac{q_v}{2\lambda}\int_0^\delta y^2\cos\frac{\pi m}{l}ydy\Big)+$$

$$+F_0\sum_{k=1}^\infty a_k\int_0^\delta \varphi_k(y)\cos\frac{\pi m}{l}ydy-$$

$$-\frac{q_v}{2\lambda}\sum_{k=1}^\infty a_k\int_0^\delta y^2\varphi_k(y)\cos\frac{\pi m}{l}ydy+$$

$$+\sum_{k=1}^\infty\sum_{n=1}^\infty a_kF_n\cos\frac{\pi n}{l}\delta\times$$

$$\times\int_0^\delta \varphi_k(y)\,\mathrm{ch}\frac{\pi n}{l}y\cos\frac{\pi m}{l}ydy=Q_m, \qquad (10)\ \text{(cont'd)}$$

where

$$Q_m=\int_0^\delta Q(y)\cos\frac{\pi m}{l}ydy,$$

and the equation for Q_0 is

$$\sum_{n=1}^\infty F_n\left(a_0\cos\frac{\pi n}{l}\delta-\frac{\pi n}{l}\sin\frac{\pi n}{l}\delta\right)\times$$

$$\times\int_0^\delta \mathrm{ch}\frac{\pi n}{l}ydy+$$

$$+a_0\left(F_0\delta-\frac{q_v\delta^3}{6\lambda}\right)+F_0\sum_{k=1}^\infty a_k\int_0^\delta\varphi_k(y)dy-$$

$$-\frac{q_v}{2\lambda}\sum_{k=1}^\infty a_k\int_0^\delta y^2\varphi_k(y)dy+$$

$$+\sum_{k=1}^\infty\sum_{n=1}^\infty a_kF_n\cos\frac{\pi n}{l}\delta\int_0^\delta\varphi_k(y)\,\mathrm{ch}\frac{\pi n}{l}ydy=$$

$$=\int_0^\delta Q(y)dy. \qquad (11)$$

After (7) was made to satisfy condition (9), and having performed the same operations, but integrating over x from 0 to l, we obtain a system of n equations of the form

$$-\frac{q_v\delta}{\lambda}\int_0^l\cos\frac{\pi n}{l}xdx+$$

$$+\sum_{n=1}^\infty F_n\left(\frac{\pi n}{l}\,\mathrm{sh}\frac{\pi n}{l}\delta+a_0\,\mathrm{ch}\frac{\pi n}{l}\delta\right)\times$$

$$\times\int_0^l\cos^2\frac{\pi n}{l}xdx+$$

$$+a_0\left(F_0-\frac{q_v\delta^2}{2\lambda}\right)\int_0^l\cos\frac{\pi n}{l}xdx+$$

$$+\left(F_0-\frac{q_v\delta^2}{2\lambda}\right)\sum_{k=1}^\infty a_k\int_0^l\varphi_k(x)\cos\frac{\pi n}{l}xdx+$$

$$+\sum_{k=1}^\infty\sum_{n=1}^\infty a_kF_n\,\mathrm{ch}\frac{\pi n}{l}\delta\int_0^l\varphi_k(x)\cos^2\frac{\pi n}{l}xdx=Q_n, \qquad (12)$$

where

$$Q_n=\int_0^l Q(x)\cos\frac{\pi n}{l}xdx,$$

and

$$-\frac{q_v\delta l}{\lambda}+\sum_{n=1}^\infty F_n\left(\frac{\pi n}{l}\,\mathrm{sh}\frac{\pi n}{l}\delta+a_0\,\mathrm{ch}\frac{\pi n}{l}\delta\right)\times$$

$$\times\int_0^l\cos\frac{\pi n}{l}xdx+a_0l\left(F_0-\frac{q_v\delta^2}{2\lambda}\right)+$$

$$+\left(F_0-\frac{q_v\delta^2}{2\lambda}\right)\sum_{k=1}^\infty a_k\int_0^l\varphi_k(x)dx+$$

$$+\sum_{k=1}^\infty\sum_{n=1}^\infty a_kF_n\,\mathrm{ch}\frac{\pi n}{l}\delta\int_0^l\varphi_k(x)\cos\frac{\pi n}{l}xdx=Q_0,$$

$$Q_0=\int_0^l Q(x)dx. \qquad (13)$$

On the strength of the symmetry with respect to the diagonal of square OB, we have: $Q(x) = Q(y)$; $Q_m = Q_n$; $\varphi_k(x) = \varphi_k(y)$; $X_n(x) = Y_m(y)$, i.e., $\cos(\pi n/l)x = \cos(\pi m/l)y$, where $m = n$.

By subtracting the system of equations (12) from the system of equations (10), and subtracting Eq. (13) from (11), we obtain a system of $(n + 1)$ equations with $(n + k + 2)$ unknowns $F_0, F_1, \ldots, F_n$; $a_0, a_1, \ldots, a_k$. Thus, to close the system of equations, $(k + 1)$ equations must be still obtained. Assuming we have measured experimentally $(n + 1)$ temperatures along the x-axis on the outer surface of the channel, then by substituting the experimental temperatures and their corresponding coordinates into (7) we obtain a system of $(n + 1)$ equations from which the $(n + 1)$ unknowns $F_0, F_1, \ldots, F_n$ can be determined:

$$\theta(x,0)=F_0+\sum_{n=1}^\infty F_n\cos\frac{\pi n}{l}x. \qquad (14)$$

If we set $n = k$, then by substituting the values of $F_0, F_1, \ldots, F_n$ into the systems of equations (10)–(12) and (11)–(14), we arrive at a system of $(n + 1)$ equations which is solvable with respect to the $(n + 1)$ unknowns $a_0, a_1, \ldots, a_k$.

Thus, the problem can be solved only in the case in which the number of terms in (5) is equal to the number of experimentally measured temperatures. The obtained coefficients $a_0, a_1, \ldots, a_k$ can be substituted into (5) to obtain the functions $h(x)$ and $h(y)$, and, also, a relation for the variation of the heat-transfer coefficient along the inner perimeter, $\alpha(x)$ and $\alpha(y)$.

Obviously, the number of experimentally measured wall temperatures will be limited in practice. Hence, in the approximate solution of the problem, the number of equations in the systems and the number of terms in the sums will be limited. The accuracy in the determination of the desired relations improves with increasing number of experimentally measured outer-surface temperatures. To improve the accuracy, it is necessary in the experimental scheme to increase the cross-sectional dimensions of the channel and to reduce the wall thickness and the junction dimensions of the thermocouples used to measure the wall temperatures.

As an example, the figure shows the wall temperature distribution obtained from (6) for a square channel under the assumption that the heat-transfer coefficients are constant along the inner perimeter.

The arbitrary constants $Q_0, Q_1, \ldots, Q_n$ in (6) were determined in the same manner as above. As infinite system of algebraic equations with an infinite number of unknowns $Q_0, Q_1, \ldots, Q_n$ was obtained. The system of equations was solved approximately for values of n up to 3. It was found that, because of the good convergence, the calculations may be limited to Q_0 and Q_1, so that the following system of two equations with two unknowns need be solved:

$$\left(\frac{\delta}{l}-1\right) Q_0 + \frac{2\left(hl\cos\frac{\pi\delta}{l}-\pi\sin\frac{\pi\delta}{l}\right)\operatorname{sh}\frac{\pi\delta}{l}}{\pi\left(\pi\operatorname{sh}\frac{\pi\delta}{l}+hl\operatorname{cn}\frac{\pi\delta}{l}\right)} Q_1 + \frac{q_v\delta^2}{\lambda}+\frac{q_v h\delta^3}{3\lambda}=0,$$

$$\frac{\sin\frac{\pi\delta}{l}}{\pi} Q_0 + \left\{\left(hl\cos\frac{\pi\delta}{l}-\pi\sin\frac{\pi\delta}{l}\right)\times\left(\operatorname{sh}\frac{\pi\delta}{l}\cos\frac{\pi\delta}{l}+\operatorname{ch}\frac{\pi\delta}{l}\sin\frac{\pi\delta}{l}\right)\times\left[\pi\left(\pi\operatorname{sh}\frac{\pi\delta}{l}+hl\operatorname{ch}\frac{\pi\delta}{l}\right)\right]^{-1}-1\right\} Q_1 + \frac{q_v l\delta}{\pi\lambda}\sin\frac{\pi\delta}{l} - \frac{q_v hl^2\delta}{\pi^2\lambda}\cos\frac{\pi\delta}{l}+\frac{q_v hl^3}{\pi^3\lambda}\sin\frac{\pi\delta}{l}=0. \qquad (15)$$

The inverse problem, i.e., the determination of the heat-transfer coefficient distribution from a given temperature distribution can be readily solved by the method described. This involves merely a slight increase in computational labor.

The described procedure for calculating heat transfer was applied to the processing of experimental data obtained for the turbulent flow of a coolant in electrically heated square channels 3.5 × 3.5 mm and 4.7 × × 4.7 mm in cross section with wall thicknesss of 0.7 mm. For single-phase flow (Re = 14 000 to 150 000), the heat-transfer coefficients were found to be practically constant along the perimeter and equal to the heat-transfer coefficients in circular tubes calculated by the equivalent-diameter technique.

Surface boiling sets in always in the corner regions, the heat-transfer coefficients increasing abruptly in these areas. During surface boiling, the heat-transfer coefficients and heat fluxes undergo pronounced changes along the perimeter, decreasing from the corner to the middle of the wall. The relation between the local heat-transfer intensity and the local thermal fluxes remains the same as in circular tubes, i.e., $\alpha m \sim q_m^{0.7}$.

NOTATION

t is the wall temperature; q_V is the specific volume heat release in the wall; q is the specific heat flux to the liquid; λ is the specific thermal conductivity of the wall material; α is the heat-transfer coefficient to the fluid; δ is the wall thickness, l is half the length of the tube outer surface; x and y are coordinates; subscript fl denotes fluid; subscript l denotes local.

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